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# Weak Decays of Heavy Quarks*

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## 1. INTRODUCTION

It is generally anticipated that at least two more quark flavors will be discovered sooner or later, and I will discuss some of the properties that may help to identify them: lifetime, branching ratios, selection rules, lepton decay spectra. In addition, there is the exciting possibility that CP violation may manifest itself more strongly in heavy particle decays than elsewhere, providing a new probe of its origin.

Predictions of these properties, however, require some understanding of the dynamics of non-leptonic transitions, and I will first try to convince the reader that theorists have made considerable progress in the understanding of non-leptonic transitions among lighter quarks. As the technology of QCD has been developing there has been a feed-back of application to the long standing problems of non-leptonic K- and hyperon-decay, and a rather staisfactory description of these decay amplitudes has emerged. Within the same framework predictions were made for the decays of the $\Omega^-$ and of charmed particles; we shall see how they compare with the data now available.

In addition to a framework for treating strong interaction effects, we need a model for the weak coupling of heavy quarks; I will restrict my discussion to the Kobayashi-Maskawa model. After a brief justification of this choice, I will go into details of its implications for topology and bottomology.

## 2. DYNAMICS OF NONLEPTONIC DECAYS

The first step is to find the effective local operators which can induce transitions among the quarks in the hadron wave function. For strangeness changing processes the most important operators are four fermion couplings. For example, the quark scattering process

$$s + u \rightarrow u + d \qquad ,$$

for all external momenta $<<m_W$ is obtained by summing diagrams of the type shown in Fig. 1. Since gluon exchange conserves helicity, the primary V-A coupling structure is unchanged, but the effective fermi coupling constant gets renormalized, the renormalization factor depending on the color representation of the scattering channel. For a V-A pointlike interaction, scattering occurs in an s-wave spin zero channel which is antisymmetric in the quarks. The color and flavor wave functions must therefore have the same symmetry properties: the final state u and d quarks will have $I = 0$ for color $\overline{3}$ scattering and $I = 1$ for color 6 scattering. Since the initial (s, u) state has $I = ½$, the $\overline{3}$ scattering amplitude is pure $\Delta I = ½$, while the 6 amplitude is a mixture of $I = ½$ and $I = 3/2$. It turns out that renormalization effects enhance the effective Fermi coupling constant in the $\overline{3}$ channel and suppress it in the 6 channel. In the leading log approximation,[1] i.e. up to $O\left(\ln(m_W{}^2/\Lambda^2)^{-1}\right)$

$$G_{F_i}^{eff} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(m_W{}^2)}\right)^{\gamma_i} G_F \tag{1}$$

$$\gamma_{\overline{3}} = -2\gamma_6 = 12/(33 - 2N_f) \tag{2}$$

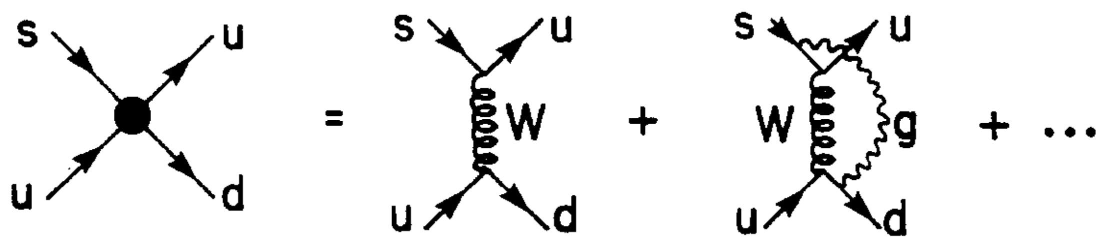


Fig. 1. Effective local $|\Delta S| = 1$ quark scattering operator

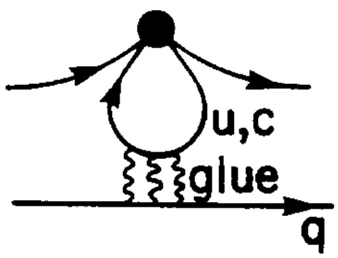


Fig. 2. Generic penguin diagram

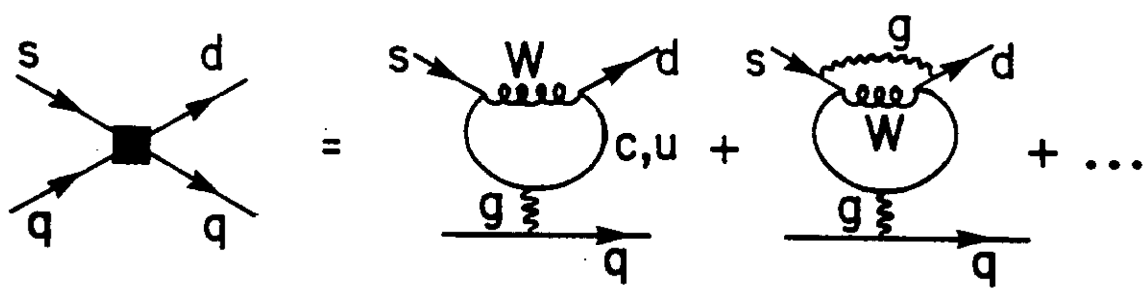


Fig. 3. Dominant effective local $|\Delta S| = 1$ operator generated by a penguin diagram.

$$\alpha_s(Q^2) = \frac{12\pi}{33 - 2N_f} \frac{1}{\ln(Q^2/\Lambda^2)} \tag{3}$$

where $m_W$ is the intermediate boson mass, $\mu$ is a typical hadron mass, O(1 GeV), $N_f$ is the number of quark flavors and $\Lambda$ should be approximately the same as the parameter measured in deep inelastic lepton-hadron scattering experiments.[2] Assuming $N_f = 6$, $\Lambda = 500$ MeV, the numerical results are

$$G_F^{eff}(\bar{3}) \simeq 2.6\ G_F$$

$$G_F^{eff}(6) \simeq 0.6\ G_F \tag{4}$$

giving a factor 4-5 enhancement of $\Delta I = 1/2$ over $\Delta I = 3/2$ amplitudes, which is not by itself sufficient to explain the observed amplitude enhancement factor of about 20.

If the u and c quarks were degenerate (and the mixing with a t quark negligible), the operators discussed above (and denoted by a black circle as in Fig. 1) would be the only operators contributing to $O(1/m_W{}^2)$. However there is a class of diagrams,[3] generically referred to as "penguin diagrams,"[4] which arise when the external u or c quarks of Fig. 1 are connected and communicate via gluon exchange with other quarks in the hadron wave function as shown in Fig. 2. All these diagrams are pure $\Delta I = ½$ because gluons cannot transmit isospin. Since these diagrams are unimportant for large internal momenta where the u, c mass difference can be neglected, their strength is characterized by $m_c{}^2$ rather than $m_W{}^2$. To leading order in $\mu^2/m_c{}^2$, the dominant effective operator is again a 4-quark operator.[5] (In the valence quark model used below the only other relevant operator is a 6-quark local operator which may have a small matrix element, and operators involving external gluons vanish for soft gluons, so the approximation of retaining only the 4-quark operator may not be too bad even for penguins.) It will be denoted by a solid square, and the effective coupling is obtained by summing diagrams of the type shown in Fig. 3. The effective quark operator is local because by gauge invariance the s-d-gluon vertex must have a $q^2$ factor which cancels the pole in the gluon propagator. Again in the leading log approximation (this time to $O\left(\ln(m_c{}^2/\Lambda^2)^{-1}\right)$, the effective Fermi coupling is given by the value of the lowest order diagram of Fig. 3 times a renormalization factor coming from the sum over extra gluon exchange:

$$G_F^{\rm eff}(\text{penguin}) \approx \ln\left(\frac{m_c{}^2}{\mu^2}\right) \frac{\alpha_s(m_c{}^2)}{6\pi}\left(\frac{\alpha(\mu^2)}{\alpha(m_c{}^2)}\right)^{2\gamma_{\bar{3}}} G_F$$

$$\approx G_F/12 \qquad . \tag{5}$$

The effective Fermi coupling is small but the structure of the operator is not of the V-A type:

$$O_{\rm penguin} = (\bar{d}\lambda_i s)_{V-A}(\bar{q}\lambda^i q)_V \tag{6}$$

where the $\lambda^i$ are color SU(3) matrices. In order to express the operator in terms of color singlet bilinears, we must perform a Fierz transformation. Writing

$$(\bar{q}\lambda^i q)_V = (\bar{q}\lambda^i q)_{V-A} + (\bar{q}\lambda^i q)_{V+A} \quad , \tag{7}$$

the (V-A)× (V-A) structure is invariant under a Fierz transformation, but

$$(\bar{d}\lambda_i s)_{V-A}(\bar{q}\lambda^i q)_{V+A} \xrightarrow[\text{Fierz}]{} (\bar{d}q)_{S+P}(\bar{q}s)_{S-P} \qquad . \tag{8}$$

For q = u, d, the bilinear $(\bar{d}q)_P$ has the quantum numbers of the pion, and this results in a considerable enhancement for certain matrix elements.

The second step is to evaluate the matrix elements of the operators obtained above. This has been done[5] assuming a simple valence quark model for the hadron wave functions. First consider baryon decays: $B \to B' + \pi_C$. The penguin operator has enhanced matrix elements only when two quark lines are attached to the external pion, and we shall neglect it elsewhere. Then the relevant amplitudes are those shown in Fig. 4. Since the baryon wave function is totally antisymmetric under color SU(3), any quark pair is in a $\bar{3}$, and only the enhanced, $\Delta I = \frac{1}{2}$ part, of the quark scattering operator contributes if two quark legs are connected to the same baryon state.[6] Therefore the diagrams 4a-4c are predominantly $\Delta I = \frac{1}{2}$; they can be evaluated in the standard soft pion treatment[7] which relates both s and p wave amplitudes to the baryon-baryon transition matrix elements shown in Fig. 5. In the non-relativistic SU(6) model these are determined in terms of a single parameter, the probability $|\psi(0)|^2$ for finding two quarks at the same point in the baryon wave function. Fig. 4d is pure $\Delta I = 1/2$; Fig. 4e is a mixture of $\Delta I = 1/2$ and 3/2. It vanishes in the chiral symmetry limit ($m_{u,d}$, $m_\pi^2 \to 0$) while Fig. 4d does not. Neglecting gluon exchange effects other than those included in the renormalized fermi coupling constants the amplitudes can be factorized in terms of matrix elements of quark bilinears:

$$M^{4e} \propto \langle B|J_\mu|B'\rangle\langle\pi|J_\mu|0\rangle \simeq f_\pi\langle B'|\partial_\mu J_\mu|B\rangle$$

$$M^{4d} \propto \frac{1}{m_s m_{u,d}}\langle B|\partial_\mu J_\mu|B'\rangle\langle\pi|\partial_\mu J_\mu|0\rangle \simeq \frac{f_\pi m_\pi^2}{m_s m_{u,d}}\langle B'|\partial_\mu J_\mu|B\rangle \qquad (9)$$

where $J_\mu$ is the usual V-A current operator and we have used the standard assumption (required in most gauge theories) that they are conserved up to quark mass terms. The pion decay constant $f_\pi$ is defined by

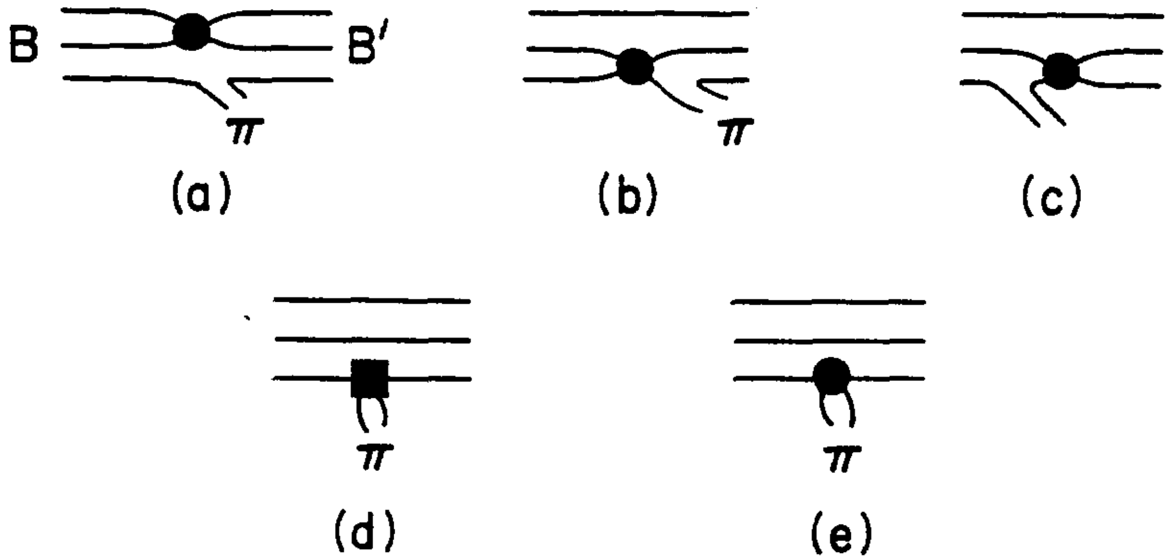


Fig. 4. Matrix elements for baryon decay: $B \to B'\pi$.

Fig. 5. Matrix element for weak baryon to baryon transition.

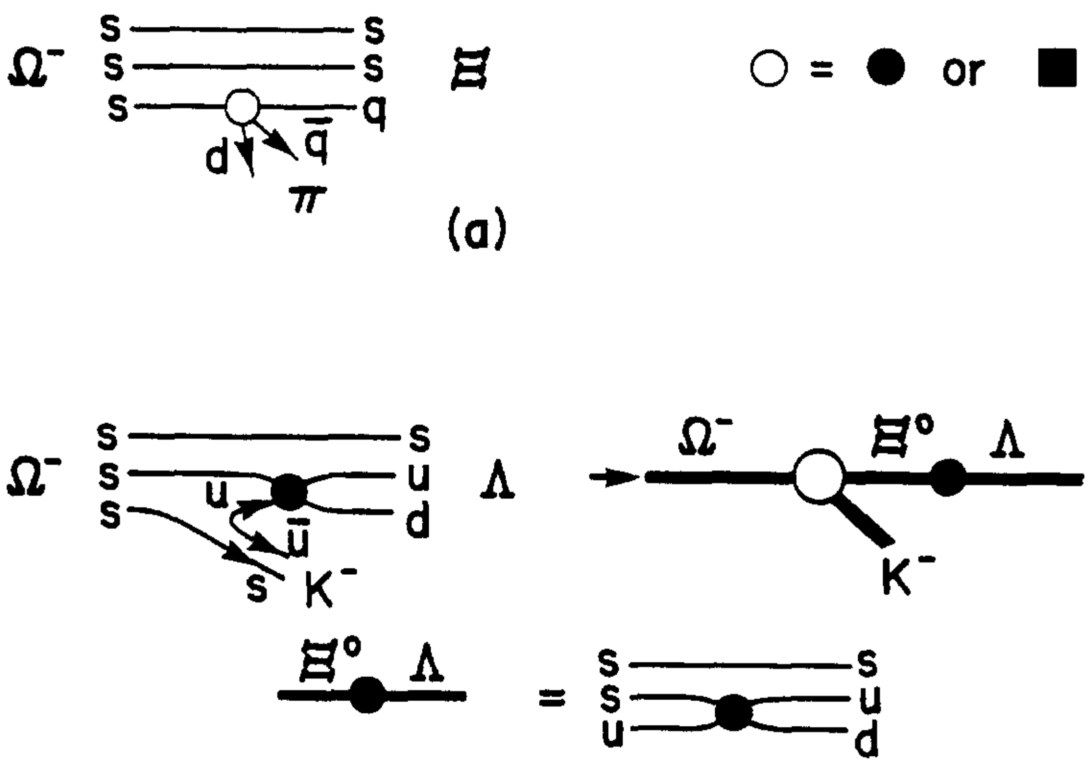


Fig. 6. Matrix elements for $\Omega^-$ decay: (a) $\Omega^- \to \Xi\pi$, (b) $\Omega^- \to \Lambda K^-$.

$$<\pi|J_\mu|0> = f_\pi p_{\pi\mu} \quad . \tag{10}$$

As a first remark we see that the $\Delta I = 3/2$ amplitudes, arising solely from Fig. 4e in this approximation are completely determined in terms of the known matrix elements relevant to semi-leptonic decays. They agree[5] with experiment in both sign and magnitude up to a common factor of about 1.5, for both K- and baryon decay (with the possible exception of $\Lambda \to p\pi$). Secondly, using conjectured values[8] for the "current quark" masses:

$$m_u \simeq m_d \simeq 5 \text{ MeV} \quad , \quad m_s \simeq 150 \text{ MeV} \tag{11}$$

we see that matrix element ratio of 4d to 4e is considerably enhanced:

$$M^{4d}/M^{4e} \simeq \frac{m_\pi^2}{m_s m_{u,d}} \simeq 26 \quad . \tag{12}$$

Absorbing this factor into the effective Fermi coupling constant Eq. (5) we get

$$\overline{G_F}^{eff}(\text{penguin}) \simeq 2.2 \quad , \tag{13}$$

a coupling comparable to the enhanced $\Delta I = ½$ part of the quark scattering operator, Eq. (4). Putting everything together, a fit to all baryon decay amplitudes can be made,[5] which determines the single unknown parameter $|\psi(0)|^2$.

Applying the above model to $\Omega^-$ decay,[9] the decay amplitudes are uniquely determined by the parameters used to fit baryon decay. In the $\Omega^-$ case the amplitudes are particularly simple. For $\Omega^- \to \Xi\pi$ only diagrams of the type 4d and 4e can contribute (see Fig. 6a) because only one strange quark can participate in the quark scattering of Fig. 1. The matrix element factorizes:

$$A(\Omega^- \to \Xi\pi) \propto \langle\Xi|J_\mu|\Omega\rangle\langle\pi|J_\mu|0\rangle$$

$$\text{or} \qquad J_\mu \to \partial_\mu J_\mu \quad .$$

In the SU(3) limit only the axial current contributes to the $\Omega$ - $\Xi$ transition at low $q^2$ (here $q^2 = m_\pi^2$), and since only the axial current contributes to the $\pi$-vacuum transition, the amplitudes are predicted to be nearly parity conserving. For $\Omega^- \to \Lambda K$ the only diagram is that of Fig. 4c. Phenomenologically, it should be dominated by the $\Xi^0$ pole diagram of Fig. 6b, because of both the proximity of the pole and the large wave function overlap for the spectator baryon. It then depends on the wave function overlap $|\psi(0)|^2$, and will again be predominantly parity conserving. The predicted rates[9] agree with the experimental value[10] within about a factor two. This is well within the theoretical uncertainties on both $|\psi(0)|^2$ and the $\Omega^- \to \Xi^-$ current matrix element. Free of these uncertainties are the predictions of vanishing asymmetry parameters

$$\alpha_{\Omega^-} \simeq 0$$

and the violation of the $\Delta I = ½$ rule[9]

$$\Gamma(\Xi^0\pi^-)/\Gamma(\Xi^-\pi^0) \simeq 3$$

which are in remarkable agreement with the experimental results[10]

$$\alpha_{K\Lambda} = 0.06 \pm 0.14$$

$$\Gamma(\Xi^0\pi^-)/\Gamma(\Xi^-\pi^0) = 2.93 \pm 0.45 \quad .$$

It should be emphasized that the large (20%) violation of the $\Delta I = \frac{1}{2}$ rule found in $\Omega^- \to \Xi\pi$ strongly supports the idea of a dynamical origin of the approximate $\Delta I = \frac{1}{2}$ rule. In the picture described here, it can be understood by the absence of the $\Delta I = \frac{1}{2}$ dominated diagrams of Figs. 4a-4c.

We turn now to meson decays. The $K \to 3\pi$ decay is successfully determined[11] by soft pion theorems from the $K \to 2\pi$ decay, so we need only consider the latter. The possible diagrams all factorize and are shown in Fig. 7. Fig. 7a gets a contribution only from the operator of Fig. 2 because the strangeness conserving part of the weak current is conserved and cannot create two pions in a zero angular momentum state:

$$\langle \pi\pi(J=0) | J_\mu | 0 \rangle = 0 \qquad .$$

Fig. 7b is given by amplitudes similar to those of Eq. (9). The operator of Fig. 1 gives an amplitude ratio $1/2 : 3/2 \simeq 4\text{-}5$, while the penguin operator of Fig. 2 gives a contribution (Eq. (13)) similar to the enhanced part of Fig. 1, so the overall $\Delta I = \frac{1}{2}$ enhancement for 7b is about a factor 10 (modulo the appropriate Clebsh Gordan factors ). Adding 7a and 7b, one finds[5] the experimental enhancement factor of 20 if one suppresses the $\Delta I = 3/2$ part by an extra factor of 1.5 as needed for baryon decays. (This could be due to extra gluon exchange effects.)

Next we turn to charm decays.[3,5,12] First we note several reasons why strong interaction effects should be weaker than for strange particle decays.

a) There are no penguins for the dominant $\Delta C = \Delta S = \pm 1$ transitions, since the basic four fermi coupling (Fig. 8) involves no identical quarks.

b) The coupling constant renormalization is weaker since the average momentum transfer is characterized by the charmed quark mass:

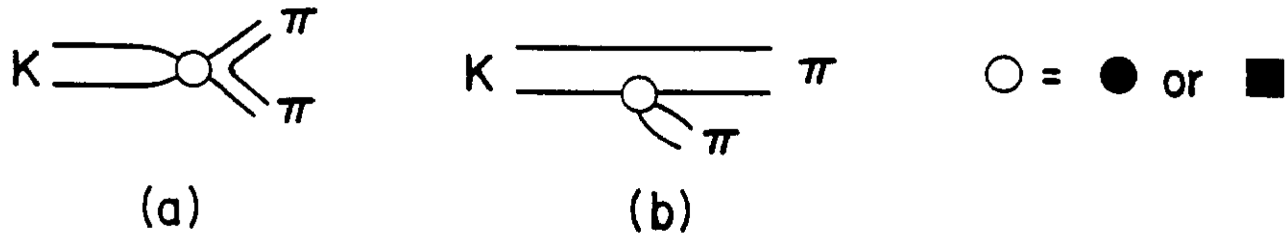


Fig. 7. Matrix elements for $K \to \pi\pi$.

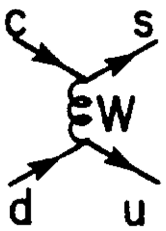


Fig. 8. Dominant $|\Delta C| = 1$ transition process.

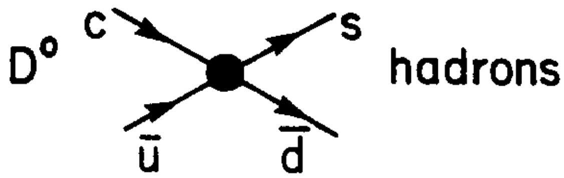


Fig. 9. Matrix element for $D^0$ annihilation decay channel.

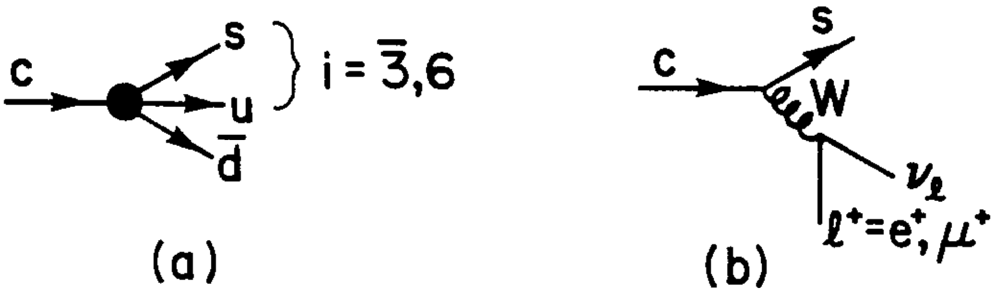


Fig. 10. Matrix elements for inclusive charm decays: (a) non-leptonic and (b) semileptonic.

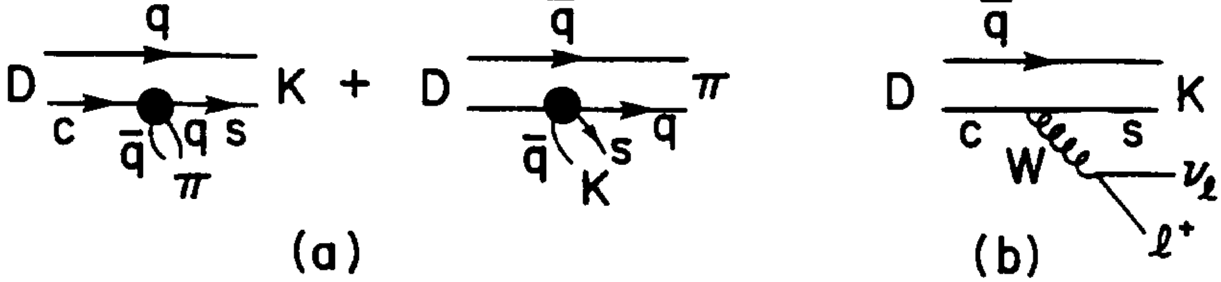


Fig. 11. Matrix elements for exclusive charm decays: (a) $D \to K\pi$, (b) $D \to K\ell^+\nu_\ell$.

$$1 - G^{eff}/G = O\left(\ln(m_W{}^2/m_c{}^2)\right) \text{ instead of } O\left(\ln(m_W{}^2/\mu^2)\right) \quad .$$

c) Because of increased phase space, there is no dynamical suppression of diagrams of the type of Fig. 7b since pion emission, which is suppressed by approximate chiral symmetry, can be replaced by $\rho$ emission, or more generally any spin-1 hadronic system with a mass $\sim 1$ GeV.

The processes which can be most readily estimated using QCD technology are inclusive decays and the exclusive channels $D \to K\ell\nu$, $D \to K\pi$. We shall neglect the contribution of Fig. 9 which has branching ratio $\approx f_D{}^2 m_s{}^2/m_c{}^4$ and should be small unless $f_D \gg f_\pi$, where $f_D$ is defined similarly to $f_\pi$, Eq. (10). Then the inclusive hadronic and leptonic decay rates, Figs. 10a and 10b, respectively, are given by*

$$\Gamma(D \to \text{hadrons}) \simeq \sum_i \left(G_i^{eff}/G_F\right)^2 \left(\frac{m_c}{m_\mu}\right)^5 \Gamma_\mu$$

$$\Gamma(D \to h + \ell\nu) = 2\left(\frac{m_c}{m_\mu}\right)^5 \Gamma_\mu \tag{14}$$

where $\Gamma_\mu$ is the muon decay rate. The total lifetime prediction

$$\tau_D \simeq (1 - 4)\, 10^{-13}$$

is sensitive to the value (1.5 - 2 GeV) used for $m_c$, but this uncertainty disappears in the total leptonic branching ratio

$$B_e = B_\mu \simeq (10\text{-}13)\%$$

where we have again used a 6-flavor model and $\Lambda = 500$ MeV to evaluate the $G_i^{eff}$. The exclusive decay amplitudes, Fig. 11, are given by*

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*Color factors for the hadronic decays are implicit.

$$A(D \to K\pi) = \sum_i G_i^{\rm eff}/\sqrt{2} < K \,|J_\mu|\, D >< \pi \,|\, J_\mu \,| 0 >$$

$$A(D \to K\ell\nu) = G_F/\sqrt{2} < K \,|J_\mu|\, D >< \ell\nu \,|\, J_\mu \,| 0 > \qquad . \qquad (15)$$

The D-K current matrix element may have large SU(4) breaking corrections. However the relative ratios for different K-$\pi$ channels are sensitive only to the $G_i^{\rm eff}$. We find

$$B(D^+ \to \bar{K}^o \pi^+)/B(D^o \to K^- \pi^+) = 0.77 \qquad ,$$

to be compared with the experimental value $0.68 \pm 0.33$. If we assume SU(4) symmetry we predict

$$B(D^o \to K^- \pi^+) \simeq (1 - 4)\% \qquad .$$

Using the experimental value of 2.2% to eliminate the uncertainty in the ratio $(m_c/m_\mu)^5/|<D\,|J\,|K>|^2$, we can predict the fraction of 3-body leptonic decays, finding:

$$B(K\ell\nu)/B(h\ell\nu) \simeq 0.44 \qquad .$$

We conclude this section with an optimistic view of our present understanding of non-leptonic decays, and turn to the decays of still heavier quarks. As the quark mass increases, the effects of strong interactions should become weaker still:

$$G_F^{\rm eff}/G_F - 1 \simeq 0\left[\ln(m_Q^2/m_W^2)\right] \to 0$$

as the quark mass approaches the W-mass. Penguin diagrams may be present, but they contain explicit factors of $\alpha_s(m_Q^2)$ which vanish with increasing quark mass. However, before discussing dynamics we must have a model for the weak couplings, which we shall first present and discuss.

## 3. THE KOBAYASHI-MASKAWA MODEL

This model[14] is a simple extension of the Cabibbo-GIM model[15,16] from four quarks to six. The charged current couplings are pure V-A and are given by

$$\mathscr{L}_{CC} = g\, W_\mu^\dagger\, \bar{q}_L \gamma_\mu \frac{\tau^-}{2} U q_L + hc \tag{16}$$

where U is a $3 \times 3$ generalized Cabibbo matrix. The phenomenological motivations for restricting our discussion to this model are by now many:

a) It incorporates CP violation in a way which is consistent with low energy phenomenology.[17-19]

b) A V-A coupling is now strongly favored[13] for the $\tau$ and its neutrino. Renormalizability of the Weinberg-Salam[20] model then requires[21] a new quark doublet (t, b) with a V-A coupling.

c) With the demise of the high-y anomaly, there is no evidence for right-handed charged couplings (e.g. a $(ub)_R$ coupling of the usual strength is ruled out).

d) There is now evidence for parity violation in neutral currents.[22,23] While the situation in atomic physics is still controversial,[24] the SLAC result[23] gives clear evidence for parity violation, removing another motivation for the introduction of right-handed couplings.

e) There are experimental limits[25,10] in the lifetime of the $B^-(b\bar{u})$, expected to be the lightest naked bottom state with a mass around 5 GeV:

$$\tau_B \gtrsim 5 \cdot 10^{-8} - 5 \cdot 10^{-9} \quad \text{if} \quad \sigma_B \gtrsim \sigma_T$$

as expected. This result argues against a new conserved quantum number associated with the b quark which has been suggested in the context of larger flavor groups than $SU(2) \times U(1)$ (unless the B decays into a lighter stable lepton). On more speculative theoretical grounds, the K-M model, as embedded in the Weinberg-Salam gauge model, provides the simplest viable possibility for the unification of strong, weak and electromagnetic interactions, namely the Georgi-Glashow SU(5) model.[26] This model has had a certain amount of phenomenological success; it predicts vanishing masses for all neutrinos, and determines the Weinberg angle to be[27,28]

$$\sin^2\theta_W = 0.20 \qquad .$$

Assuming there are only 6 quarks, the "constituent" masses (roughly defined as half the threshold mass for production of the corresponding naked flavor) have been estimated to be[28]

$$m_b \simeq (4.8 - 5.6) \text{ GeV}$$

$$m_s \simeq (380 - 500) \text{ MeV} \qquad .$$

What concerns us here are the charged current couplings as defined by Eq. (16). The U matrix acts between the quark vectors

$$q_L \to \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad , \qquad \bar{q}_L \to (\bar{u}\ \bar{c}\ \bar{t})_L \tag{17}$$

and can be written explicitly as

$$U = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{i\delta} & c_1c_2s_3 - s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 - c_2s_3e^{i\delta} & c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix} . \qquad (18)$$

where $s_i \equiv \sin\theta_i$, $c_i = \cos\theta_i$. If $s_i << 1$ for all the mixing angles, the matrix (18) simplifies to

$$U \simeq \begin{pmatrix} 1 & s_1 & s_1s_3 \\ -s_1 & 1 & s_3 - s_2e^{i\delta} \\ -s_1s_2 & s_2 - s_3e^{i\delta} & 1 \end{pmatrix} . \qquad (19)$$

In the limit where the t and b quarks decouple, $s_2, s_3 \to 0$, we recover the Cabibbo-GIM matrix with $s_1 = \sin\theta_C$.

Are there any empirical limits on $s_2$ and $s_3$? The experimental verification of Cabibbo universality for the ud and us couplings:

$$c_1{}^2 + s_1{}^2c_3{}^2 \simeq 1 \qquad (20)$$

forbids $s_3{}^2$ to be too large. Taking into account the experimental errors[29] on the relation (20), one gets the constraint[19]

$$s_3 \lesssim s_1 \simeq \sin\theta_C \simeq 0.23 \qquad . \qquad (21)$$

A constraint on $s_2$ is provided by the $K_L$-$K_S$ mass difference which receives a contribution from top quark exchange:

$$\frac{\Delta m_K}{m_K} \propto s_1^2 \left[ m_c^2 + s_2^4 m_t^2 + 2\frac{s_2^2 m_t^2 m_c^2}{m_t^2 - m_c^2} \ln \frac{m_t^2}{m_c^2} \right] \quad . \tag{22}$$

Since the exchange of a charmed quark of mass 1.5-2 GeV accounts by itself ($s_1^2 = s_c^2$, $s_2^2 = 0$ in Eq. (22)) for the observed mass difference, the top quark contribution cannot be arbitrarily large. Assuming it to be no larger than the charm contribution gives[30]

$$s_2 \lesssim 0.36 \tag{23}$$

if $m_t > 8$ GeV as suggested by dimuon data.[31] In the 6-quark model CP violation is described by the single parameter $\delta$. From the analysis of CP violation in the kaon system, one finds[30]

$$\frac{1}{2}\left|\frac{\mathrm{Im}\ m_K}{\Delta m_K}\right| \simeq 10^{-3} \simeq s_2 s_3 \sin\delta\ f(m_t^2/m_c^2, s_2^2) \quad . \tag{24}$$

Since $s_2$ and $s_3$ are bound from above, Eq. (24) bounds $\delta$ from below, but the bound is very weak:

$$\delta \gtrsim 10^{-3} \qquad \text{for} \qquad 8\ \text{GeV} \lesssim m_t \lesssim m_W \tag{25}$$

However, an arbitrarily large value of the parameter $\delta$ is permitted by present phenomenology.

## 4. VERY HEAVY QUARK DECAYS

We now turn to the analysis of naked top and bottom decays. To simplify the discussion we

a) ignore the renormalization of the effective coupling constant for the $\Delta B = \pm 1$ analogue of the operator in Fig. 1, since this effect is small. Using the same parameters as in section 2, we find

$$G_F^{\text{eff}}/G_F \simeq 1.4 \quad \text{and} \quad 0.85$$

for the $\bar{3}$ and 6 channels, respectively;

b) characterize the mixing parameters $s_2$ and $s_3$ by a common parameter s, expected to be no larger than $s_c \equiv \sin\theta_c \simeq 0.2$, and discount the possibility of a strong cancellation between $s_2$ and $s_3$ in the elements $s_3 - s_2 e^{i\delta}$, $s_2 - s_3 e^{i\delta}$ in the matrix (19);

c) ignore phases.

Then the mixing matrix (19) is of the approximate form:

$$U \simeq \begin{pmatrix} 1 & s_c & s_c s \\ s_c & 1 & s \\ s_c s & s & 1 \end{pmatrix} \quad . \tag{26}$$

We further assume that the $\Upsilon(9.4)$ is a $b\bar{b}$ bound state, so that

$$m_t > m_b \simeq 5 \text{ GeV} \quad . \tag{27}$$

We then obtain immediately a prediction for the relative strengths of different flavor changes in heavy quark decays:

$$\Gamma(t \to b)/\Gamma(t \to s)/\Gamma(t \to d) \simeq F\left(m_b{}^2/m_t{}^2\right)/s^2/s^2 s_c{}^2$$

$$\Gamma(b \to c)/\Gamma(b \to u) \simeq s^2 F\left(m_c{}^2/m_b{}^2\right)/s^2 s_c{}^2 \simeq \frac{1}{3s_c{}^2} \simeq 6 \tag{28}$$

where F(x) is the V-A phase space factor for the decay of a fermion of mass m into a fermion of mass xm and two massless fermions. The $t \to b$ branching ratio is very sensitive to the top quark mass; we find

$$F\left(m_b{}^2/m_t{}^2\right) \simeq \begin{cases} 0.05 \\ 1 \end{cases} \text{if } m_t = \begin{cases} 8 \text{ GeV} \\ m_W \end{cases} \quad . \tag{29}$$

But since we expect $s^2 \lesssim s_c{}^2 \simeq 0.06$, we expect in any case a significant $t \to b$ branching ratio:

$$\Gamma(t \to b) \gtrsim \Gamma(t \to s) \simeq 20\,\Gamma(t \to d) \quad . \tag{30}$$

Since the b-quark is expected to decay predominantly into charm, we anticipate spectacular multilepton events, for example:

$$\begin{array}{l} t \to b + (\text{hadrons or } \ell^+\nu_\ell) \\ \quad\;\; \hookrightarrow c + (\text{hadrons or } \ell^-\bar{\nu}_\ell) \\ \quad\quad\quad \hookrightarrow s + (\text{hadrons or } \ell^+\nu_\ell) \end{array}$$

with a (20-40)% probability for lepton emission at each step. The leptons will be characterized by a high transverse momentum; if the average decay c.m. lepton energy is a third of the energy release we find

$$\langle p_{\perp} \rangle \simeq \langle E_{\ell} \rangle_{c.m.} \simeq Q/3 \simeq \begin{cases} 400 \text{ MeV} & c \to s\ell\nu \\ 1 \text{ GeV} & B \to c\ell\nu \\ 3 \text{ GeV} & \Upsilon(9 \text{ GeV}) \to s\ell\nu \\ 2 \text{ GeV} & \Upsilon(11 \text{ GeV}) \to b\ell\nu \end{cases} .$$

(For charm decay we would naively predict $\langle E_{\ell} \rangle_{c.m.} \sim m_c/3 \sim$ (500-600) MeV. The observed value of about 400 MeV may be attributable to gluon bremsstrahlung[32] which should be less important for higher mass systems.)

Since the specifics of heavy quark decays are highly mass dependent, we shall hereafter concentrate on b-decay, under the assumptions (27). In Figs. 12 we show[33] the lepton spectra for the process

$$e^+e^- \to \bar{b}b \to \text{hadrons}$$

at a c.m. energy of 20 GeV, assuming an elementary 4-fermion V-A coupling for the decay, and under several assumptions for the quark fragmentation functions. While the precise shape of the spectra are model dependent, their qualitative features are not and the leptons originating at the $b \to c$ vertex (primary) and $c \to s$ vertex (secondary) appear to be separable. Fig. 13 shows transverse momentum distributions using different models for the decay processes. Again the primary and secondary leptons appear separable.

Aside from observing multi-lepton events and measuring lepton decay spectra, we may hope to study final state quantum numbers and look for particular final state configurations such as[34] two-jet decay channels. The basic decay mechanisms are shown in Fig. 14, and the corresponding final state characteristics and estimated branching ratios[30] are given in Table 1. Figs. 14a, b show the dominant free 3-body quark decay mechanism dominated by charmed final states as discussed above, Eq. (28).

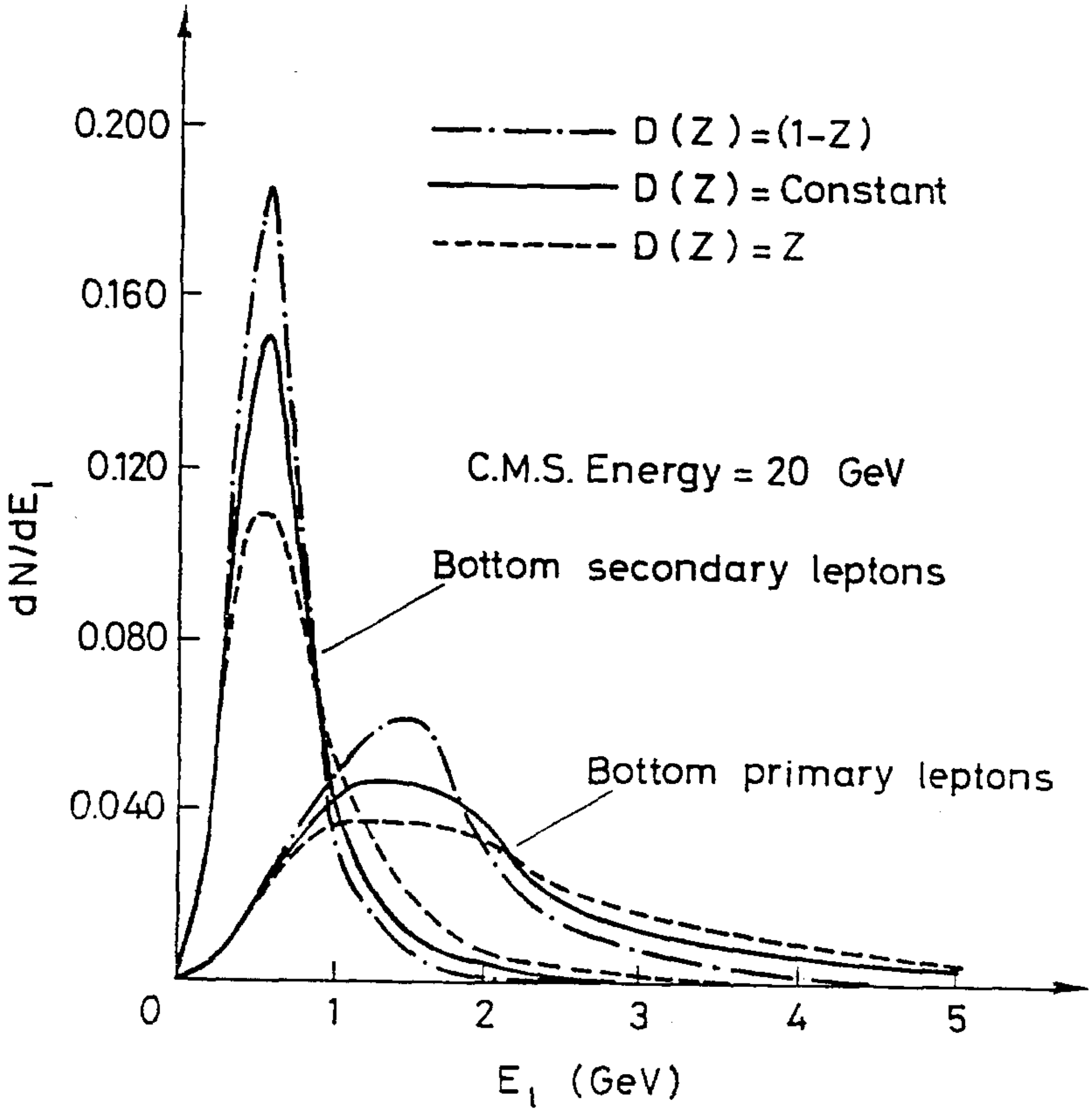



Fig. 12. Energy spectra[33] for primary and secondary leptons from bottom decay in the reaction $e^+e^- \to B^0\bar{B}^0 + X$ at $\sqrt{s} = 20$ GeV assuming $m_B = 5.1$ GeV, $m_c = 2.0$ GeV, $m_s = 0.7$ GeV, and a V-A four fermion interaction, for different assumptions on the quark fragmentation functions.

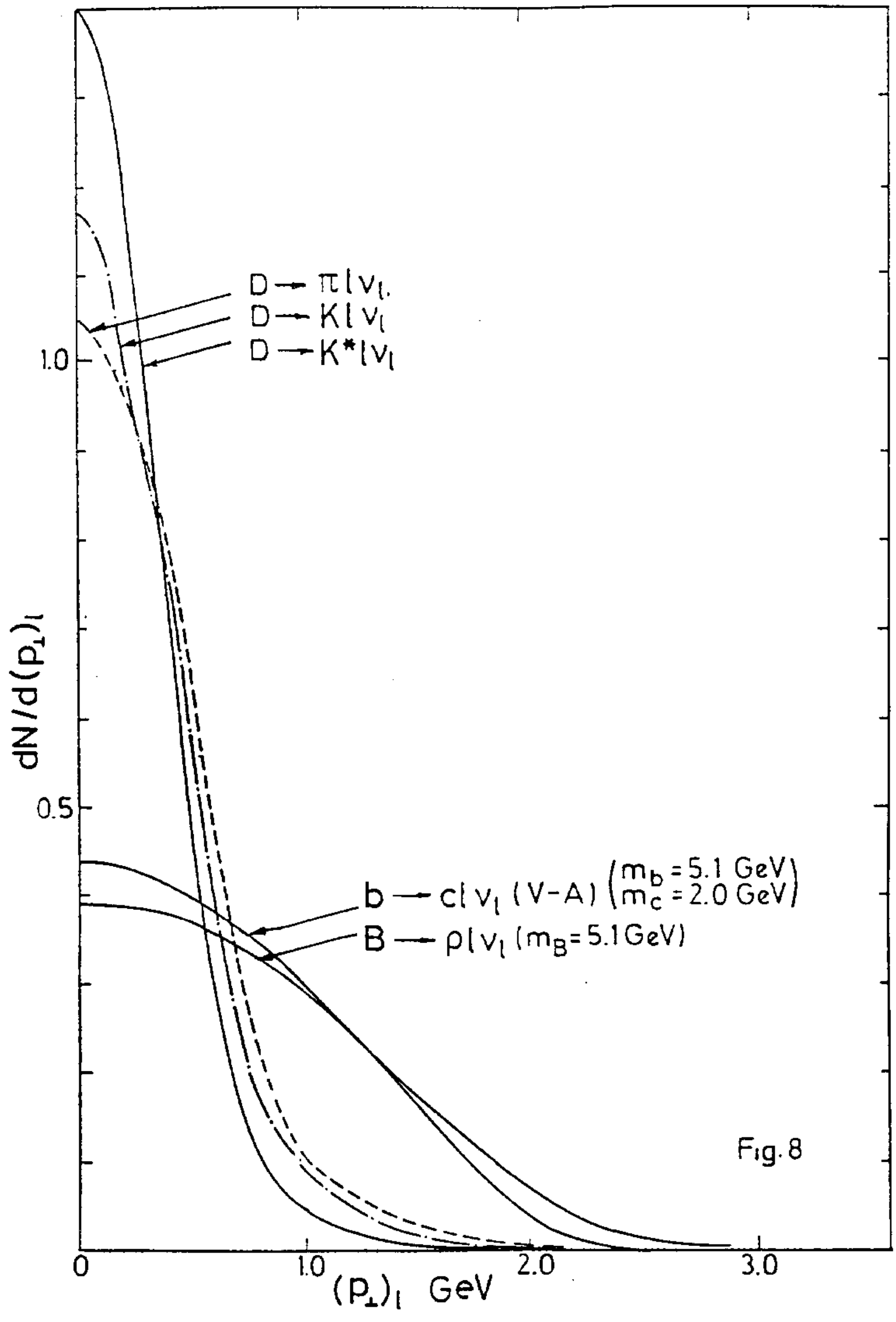



Fig. 13. Transverse lepton distributions[33] for secondary and primary leptons from bottom decay under different assumptions for decay mechanisms.

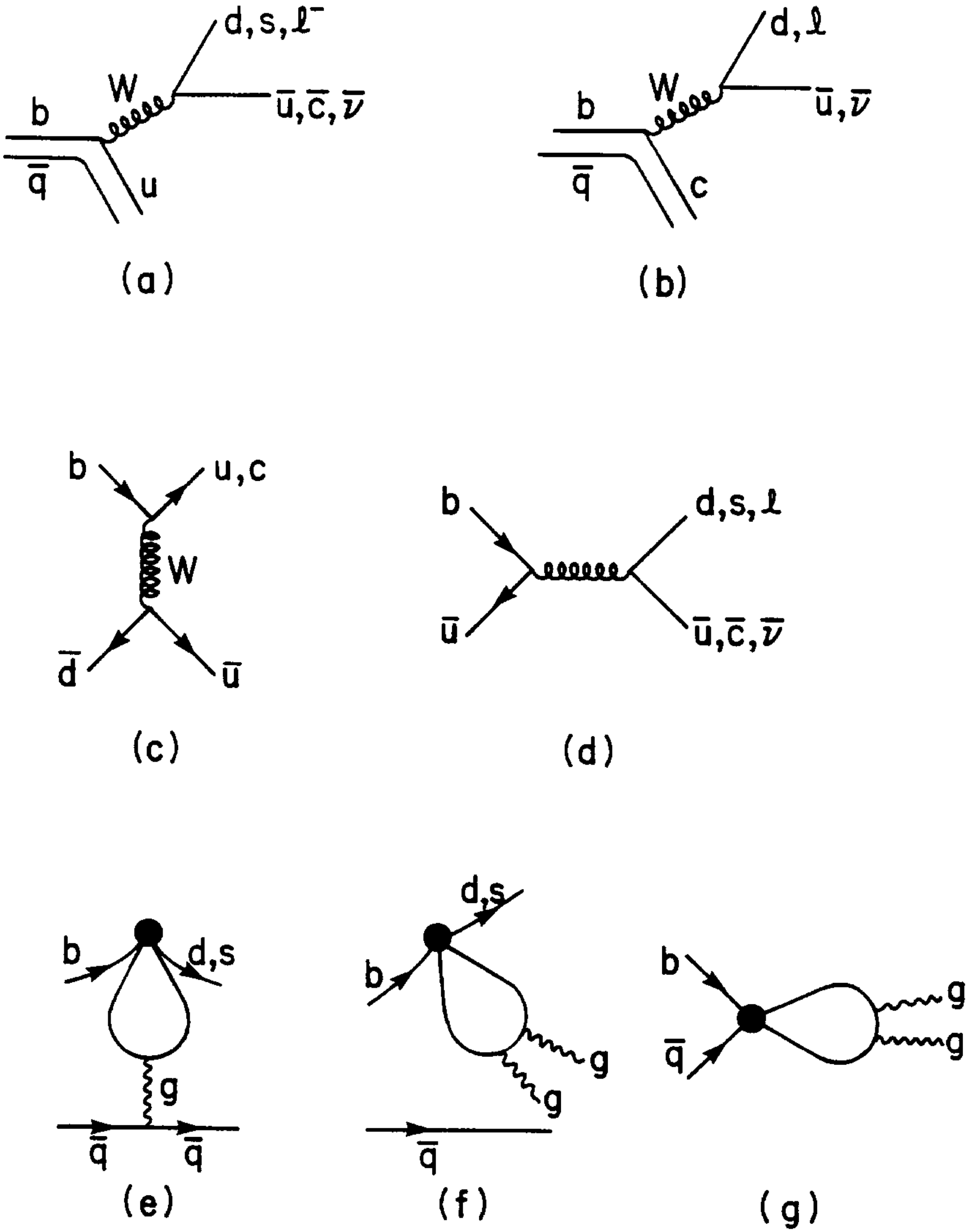



Fig. 14. Diagrams for $B(b\bar{q})$ decay.

Table 1

Mechanisms for bottom decay

| Diagram (Fig. 14) | Final State | $\left(\frac{\text{Amplitude}}{G_F s}\right)^2$ | Branching Ratio (%) |
|---|---|---|---|
| (a) | $B \to$ hadrons | $3s_c^2 \simeq 0.15$ | 7 |
| | $B \to \bar{c}$ + hadrons | $s_c^2 \simeq 0.05$ | 2 |
| | $B \to \ell^- \bar{\nu}_\ell$ + hadrons | $2s_c^2 \simeq 0.10$ | 5 |
| (b) | $B \to c$ + hadrons | $3 \times \frac{1}{3} \simeq 1$ | 45 |
| | $B \to c + \ell^- \nu_\ell$ + hadrons | $2 \times \frac{1}{3} \simeq \frac{2}{3}$ | 30 |
| (c) | $B^0 \to$ 2 jets | $\frac{f_B^2 m_u^2 s_c^2}{m_b^4} \sim 10^{-4}$ | - |
| | $B^0 \to c$ + jet | $f_B^2 \frac{m_c^2}{m_b^4}\left(1 - \frac{m_c^2}{m_b^2}\right)^2 \sim 0.10$ | 5 |
| (d) | $B^- \to$ 2 jets | $10^{-3}$ | - |
| | $B^- \to c$ + jet | $s_c^2 \simeq 0.05$ | 2 |
| | $B^- \to \ell\nu$ | $\sim 0$ | - |
| | $B^- \to \tau\nu$ | 0.02 | 1 |
| (e) | $B \to$ 2 jets | $\left[\frac{\alpha_s(m_b^2)}{\pi} \ln\left(\frac{m_t^2}{m_b^2}\right)\right]^2 \lesssim .13$ | <6 |
| (f) | $B \to$ hadrons $(\Delta S \neq 0)$ | $10^{-3}$ | - |
| (g) | $B \to$ 2 fat jets $(\Delta S \neq 0)$ | | |

Figs. 14c, d are annihilation processes which can be more important than for charm decay (cf. Fig. 9) because of the relatively heavy charmed quark which suffers little helicity suppression and because the decay constant $f_B$ (analogue of $f_\pi$, Eq. (10)) may be large. Various estimates[34,35] suggest

$$f_B \simeq 500 \text{ MeV} \qquad . \tag{31}$$

Figs. 14e-g are the penguin diagrams. Fig. 14g is the bottom-changing analogue of the 4-quark operator of Fig. 3. Its importance depends on the t-quark mass, i.e. on the effectiveness of the generalized GIM cancellation[16] involving t-quark exchange. In any case, there is an explicit factor $\alpha_s(m_b^2)/\pi$ for these diagrams, and their contribution is not expected to exceed several percent of the total decay width. While operators containing external gluons are negligible in the valence quark model used to describe exclusive $|\Delta S| = 1$ decay channels, they need not be a negligible contribution to inclusive decays of a heavy quark. However explicit calculations[30] suggest their contribution is quite small.

Adding up the contributions of table 1, one expects a total branching ratio into charmed particles of (80-85)% and a total semi-leptonic branching ratio of about 35%. Depending on the top quark mass, one can expect a 2-jet configuration in the final state (including one fattish charmed jet) at a level of 5 or 10 percent. In addition there should be[30] $B^- \to \tau^- \nu_c$ decays at a level of about one percent, and semi-leptonic decays into a $(\tau\nu_e)$ pair at a similar level.

The total lifetime is estimated to be[30]

$$\tau_B \simeq 4 \times 10^{-15}(s_2^{\,2} + s_3^{\,2} + 2s_2s_3\cos\delta)^{-1} \qquad . \tag{32}$$

Using the upper bounds on $s_3$ and $s_2$ Eqs. (21) and (23), we obtain a lower limit on the lifetime:

$$\tau_B \gtrsim 10^{-13} \quad . \tag{33}$$

The upper limit obtained by exploiting (21) and (23)-(25) is one or two order of magnitudes longer.

The above analysis applies to the pseudoscalar $B^-(b\bar{u})$ and $B^0(b\bar{d})$ states which are expected to be the lightest naked bottom states. A similar analysis can be carried out for the strange state $B_s(b\bar{s})$, which should be very close in mass to the $B^0$, $B^-$ doublet.

## 5. MASS MIXING AND CP VIOLATION

Second order weak interaction effects induce the flavor changing ($|\Delta F| = 2$) transitions responsible for neutral particle mixing: $K^0 \leftrightarrow \bar{K}^0$, $D^0 \leftrightarrow \bar{D}^0$, $B^0 \leftrightarrow \bar{B}^0$, etc. Just as in the cases of non-leptonic decays the effective local quark operator can be derived[36] from the (gluon radiative-corrected[37]) quark scattering diagrams as indicated in Fig. 15. The leading operator is again a V-A four fermion operator, and in the valence quark approximation for meson wave functions its matrix element factorizes:

$$\Delta m_P \propto \langle \bar{P}^0 | J_\mu | 0 \rangle \langle 0 | J_\mu | P^0 \rangle = f_P{}^2 m_P{}^2$$

$$P = K, D, B, \ldots \quad . \tag{34}$$

However in the K-M model the effective Fermi coupling constant is in general complex; the strength of the amplitudes of Fig. 15 determines the amount of mass mixing, while their imaginary part governs the CP violation.

Mass mixing and CP violation in the $B^o\overline{B}^o$ system might be measurable by studying dilepton production in the process

$$e^+e^- \to B^o\overline{B}^o$$

near threshold. Mass mixing will produce "same sign" dilepton pairs:

$$\begin{array}{l} e^+e^- \to B^o\overline{B}^o + X \\ \qquad\qquad\quad \overline{B}^o \hookrightarrow \bar{c}\ell^+\nu_e \\ \qquad\qquad\quad B^o \hookrightarrow \overline{B}^o \to \bar{c}\ell^+\nu_e \end{array} \quad .$$

and CP violation would produce a charge asymetry:

$$N(\ell^+\ell^+) \neq N(\ell^-\ell^-) \quad .$$

However if B decays predominantly into charm as anticipated, a same sign dilepton background would arise from cascade decays:

$$\begin{array}{l} e^+e^- \to B^o\overline{B}^o \\ \qquad\qquad \overline{B}^o \hookrightarrow \bar{c}\ell^+\nu_e \\ \qquad\qquad\qquad \bar{c} \hookrightarrow \text{hadrons} \\ \qquad\qquad B^o \hookrightarrow c + \text{hadrons} \\ \qquad\qquad\qquad c \hookrightarrow \ell^+\nu_e + \text{hadrons} \end{array} \quad .$$

The determination of the mass mixing will then depend on the feasibility of separating primary and secondary leptons as discussed above (Fig. 12); CP violating effects will be measurable only to the extent that the mass mixing is appreciable.

This may indeed be the case, in contrast to $D^0 - \bar{D}^0$ mixing which is predicted to be negligible. In terms of the same sign dilepton rate, the mixing parameter can be expressed as[38]

$$r_1 = \frac{\sqrt{N^{++}N^{--}}}{N^{+-}} = \frac{\Delta}{1+\Delta}$$

$$\Delta = \frac{(\Delta\Gamma)^2/4 + (\Delta m)^2}{2\Gamma + (\Delta m)^2 - (\Delta\Gamma)^2/4} \tag{35}$$

where if $m_{1,2}$ and $\Gamma_{1,2}$ are the physical mass and width of the decay eigenstates:

$$\Delta m = m_1 - m_2 \quad , \quad \Delta\Gamma = \Gamma_1 - \Gamma_2 \quad , \quad 2\Gamma = \Gamma_1 + \Gamma_2 \quad . \tag{36}$$

We see that the effect will be important if $\Delta m/\Gamma$ and/or $\Delta\Gamma/\Gamma$ is large. For the neutral kaon system, mixing is maximal because both the total decay rate and the mixing amplitude (Fig. 16a) are characterized by small angles:

$$\Gamma_K, \Delta\Gamma_K \propto s_c^2 \quad , \quad \Delta m_K \propto s_c^2 m_c^2 \; , \; s_c^2 s^4 m_t^2, \tag{37}$$

and because the GIM mechanism which acts to suppress mixing is badly broken by the c-u mass difference. In fact it is totally ineffective in suppressing $\Delta\Gamma$ since charmed final states are energetically forbidden; the nonleptonic decay modes carry no net flavor and are common to both $K^0$ and $\bar{K}^0$ decay. In contrast, charm decay is not suppressed by a small angle while the mixing parameters are: the decay modes common to $D^0$ and $\bar{D}^0$ are the Cabibbo suppressed uncharmed ones. In addition, to the extent that the bottom quark coupling can be neglected ($s^2 << 1$), the GIM cancellation is more effective, broken only by the s-d mass difference (Fig. 16b)

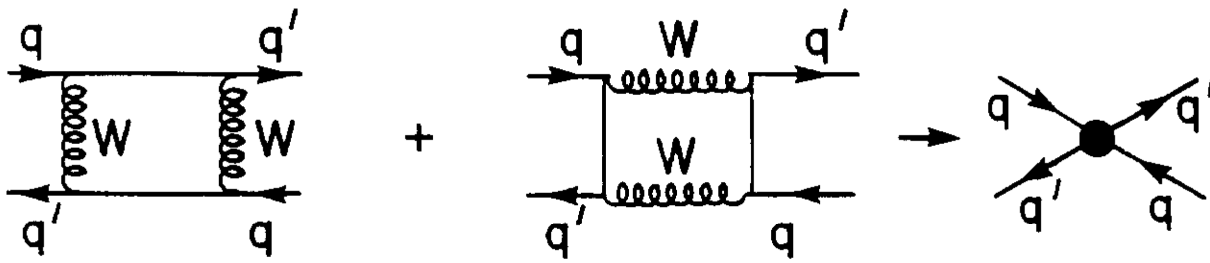



Fig. 15. Effective local operator for $|\Delta F| = 2$ transitions.

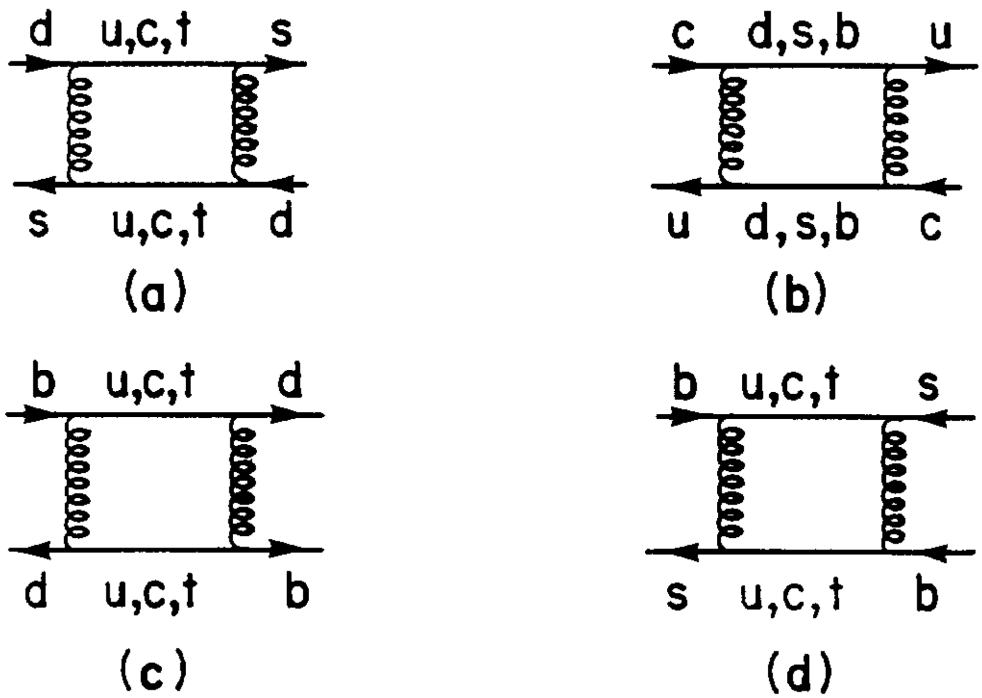
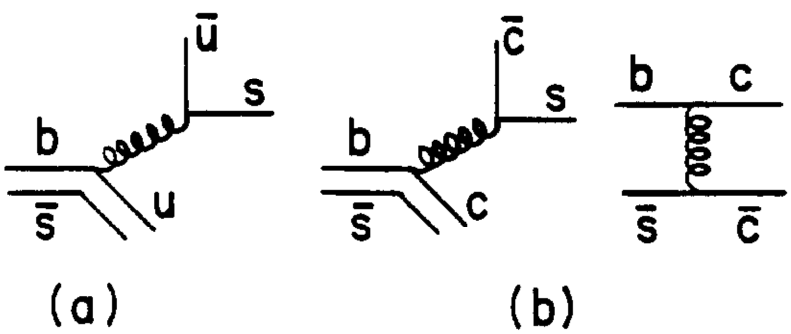



Fig. 16. Diagrams contributing to (a) $K^o - \bar{K}^o$, (b) $D^o - \bar{D}^o$, (c) $B^o - \bar{B}^o$, and (d) $B_s{}^o - \bar{B}_s{}^o$ mixing.



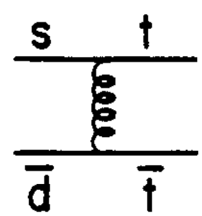




Fig. 17. $B_s$ decays into flavor neutral final states which are suppressed by (a) angles or (b) phase space.

Fig. 18. Zweig suppressed CP violating contribution to $|\Delta S| = 1$ decay amplitudes.

$$\Gamma_D \propto 1, \; \Delta\Gamma_D \propto s_c^{\,2}, \Delta m_D \approx s_c^{\,2} m_s^{\,4}/m_c^{\,2}, \; s_c^{\,2} s^4 m_b^{\,2} \quad . \tag{38}$$

For the neutral B-system, the situation is more analogous to the kaon case; the decay rates are suppressed by the same small angles as the mass mixing (Fig. 16c), or by phase space for the favored decay into charm. $\Delta\Gamma$ arises only from non-charmed final states, and so should not be too large:

$$\Gamma_B \propto s_c^{\,2} s^2 \,, \tfrac{1}{3} s^2 \quad ; \quad \Delta\Gamma_B \propto s_c^{\,2} s^2 \quad ; \quad \Delta m_B \propto s_c^{\,2} s^2 m_t^{\,2} \quad . \tag{39}$$

For the strange neutral bottom state $B_s(b\bar{s})$, mixing will be enhanced even further since the Cabibbo suppression of b-s mixing (Fig. 16d) is weaker than for b-u mixing, while Cabibbo favored decay channels are still suppressed by phase space. $\Delta\Gamma$ will be very small, since the final states common to $B_s^{\,o}$ and $\bar{B}_s^{\,o}$ are highly suppressed by angles (Fig. 17a) or by phase space (Fig. 17b):

$$\Gamma_{B_s} \propto s_c^{\,2} s^2, \; \tfrac{1}{3} s^2 \quad ; \quad \Delta\Gamma_{B_s} \propto s_c^{\,4} s^2 \quad ; \quad \Delta m_{B_s} \propto s^2 m_t^{\,2} \quad . \tag{40}$$

For the kaon system, the measured mixing parameters are

$$\frac{\Delta m_K}{\Gamma_K} \simeq \frac{1}{2} \quad , \quad \frac{\Delta\Gamma_K}{\Gamma_K} \simeq 1 \quad . \tag{41}$$

The measured value of $\Delta m_K$ agrees in sign and magnitude with the value calculated[36] neglecting top exchange if $m_c \simeq 1.5$ GeV. For neutral heavy quark systems, the predictions obtained from the analysis of the mixing (Fig. 16) and decay (e.g. Figs. 14 and 17) amplitudes are

$$\frac{\Delta m_c}{\Gamma_c} \approx \frac{\Delta \Gamma_c}{\Gamma_c} \propto 10^{-4} \tag{42}$$

for the $D^o - \bar{D}^o$ system,[39,19]

$$\frac{\Delta m_B}{\Gamma_B} \approx \left(\frac{m_t}{26\text{ GeV}}\right)^2 \gtrsim 0.1 \quad , \frac{\Delta \Gamma_B}{\Gamma_B} \approx (0.10 - 0.15) \tag{43}$$

for the $B^o(b\bar{d})$ system,[30] and

$$\frac{\Delta m_{B_s}}{\Gamma_{B_s}} \approx \left(\frac{m_t}{6\text{ GeV}}\right)^2 > 1 \quad , \quad \frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} \sim 10^{-2} \quad . \tag{44}$$

for the $B_s{}^o(b\bar{s})$ system.[40]

The six quark model was introduced by Kobayashi and Maskawa in 1973 as a mechanism for incorporating CP violation into the standard Weinberg-Salam-GIM model.[20,16] Their observation[14,18] was the following. For a theory with n weak isospin quark doublets, the mixing matrix U will be an $n \times n$ unitary matrix which is specified by $n^2$ real parameters. Of these, $(n^2 - n)/2$ define a real orthogonal matrix, so there will be $(n^2 + n)/2$ phases. However since all couplings in the theory are flavor diagonal except for the charged current coupling, Eq. (16), the matrix U can be redefined by any flavor diagonal phase transofrmation

$$u \to e^{i\alpha_u} u \quad , \quad d \to e^{i\alpha_d} d, \quad \text{etc.}$$

which leaves invariant the remainder of the Lagrangian. Any phase which can be removed from U by such a transformation is unobservable. There are a total of 2n quark flavors, and therefore 2n independent invariant phase transformations.

However a phase common to all the charge 2/3 quarks has the same effect on U as a phase common to all the charge -1/3 quarks, so there are 2n - 1 independent transformations which can be made to redefine U, leaving as the number of observable phases:

$$\frac{n^2-3n+2}{2} = \begin{cases} 0 & n=2 \\ 1 & n=3 \\ >1 & n>3 \end{cases} \quad . \tag{45}$$

There is no observable phase in the 4-quark model, while CP violation in the KM model is uniquely specified[41] by a single phase parameter. It will vanish in the limit where

a) any quark pair decouples, since then the mixing matrix reduces to the $2\times 2$ case, and

b) any two quarks of the same charge are degenerate in mass, since then there is an extra invariance which can be exploited to remove the remaining phase.

For low energy CP violation phenomenology, the model mimics the super weak model. In order for CP violation to occur, the mixing of the light quarks to the heavy (t, b) doublet has to play a role. For lowest order $|\Delta S| = 1$ decay amplitudes, CP violation will depend on the highly Zweig suppressed component of a $(t\bar{t})$ sea in the hadron wave functions, and the CP violating amplitude, Fig. 18, is purely $\Delta I = \frac{1}{2}$. CP violation in higher order weak transitions as in Fig. 16a arise from virtual top exchange. While it vanishes in the limit of quark mass degeneracy, $\Delta m_q^{\;2}/m_W^{\;2} \to 0$, the large top mass splitting makes CP violation in the kaon mass matrix the dominant effect for transitions among light quarks. In particular the neutron dipole moment is predicted to be even smaller than in the super weak model.[19,42] In all processes involving light quarks, CP violating amplitudes are characterized by the suppression factor

$$s_2{}^2 s_3{}^2 \sin^2\delta \qquad , \tag{46}$$

which knows about the coupling to heavy quarks as well as the CP violating phase.

CP violation in the $B^0\overline{B}^0$ system can be much larger than in the $K^0\overline{K}^0$ system. For kaons, the dominant contribution to $|\Delta m_K|$ comes from u, t exchange, $\propto \sin^2\theta_C$, while the top quark contribution, necessary to generate a phase is suppressed by the additional factor (46). On the other hand, for the $B^0(b\bar{d})$ system, the contributions from u, c, and t exchange are all of order $s_c{}^2 s^2$, so the system "knows" maximally about the full quark mixing. If we define the complex parameters $\Delta\bar{m}_P$ and $\Delta\bar{\Gamma}_P$ respectively as the dispersive (virtual intermediate states as in Fig. 16) and absorptive (real intermediate states) of the $P^0 \leftrightarrow \bar{P}^0$ mixing amplitude:

$$A(P^0 \leftrightarrow \bar{P}^0) = \Delta\bar{m}_P - i\frac{\Delta\bar{\Gamma}_P}{2} \qquad , \tag{47}$$

the CP violating charge asymmetry in same sign dilepton events in $e^+e^-$ annihilation can be expressed as[38]

$$r_2 = \sqrt{\frac{N^{--}}{N^{++}}} = \left| \frac{\Delta\bar{m} - i\Delta\bar{\Gamma}/2}{\Delta\bar{m}^* - i\Delta\bar{\Gamma}^*/2} \right| \qquad . \tag{48}$$

If CP violation is present, $r_2$ can differ from unity. The effect will vanish if $\Delta m$ and $\Delta\Gamma$ have the same phase, but also if $|\Delta m/\Delta\Gamma|$ or $|\Delta\Gamma/\Delta m| << 1$. The effect is therefore maximal if $|\Delta m|$ and $|\Delta\Gamma|$ are comparable. In order for it to be measurable, there must be an appreciable same sign dilepton rate; $r_1$ in Eq. (35) cannot be too small.

For the $B^0(b\bar{d})$ system, if $m_t \simeq 8$ GeV, we find

$$|\Delta m_B| \sim \left|\frac{\Delta\Gamma_B}{2}\right| \sim 0.1\ \Gamma_B$$

$$r_1 \sim 10^{-2} \quad , \quad r_2 = \frac{1 + \sin\delta}{1 - \sin\delta} \quad . \tag{49}$$

As $m_t$ increases, $|\Delta m_B|$ increases relative to $\Gamma_B$ and $|\Delta\Gamma_B|$, so the mixing gets more important but the CP violation decreases for fixed $\delta$. For $m_t \gg 8$ GeV:

$$|\Delta\Gamma_B| \ll |\Delta m_B| \sim \Gamma_B$$

$$r_1 = O(1) \quad , \quad r_2 \simeq 1 - 2\sin\delta\left(\frac{8\ \mathrm{GeV}}{m_t}\right)^2 \quad . \tag{50}$$

Since $\delta$ is arbitrary, there are at least some values of $m_t$ where these effects may be large enough to be measured. For the $B_s(b\bar{s})$ system[40] the mixing is expected to be large, but the CP violating effects are expected to be small:

$$|\Delta m_{B_s}| \gtrsim \Gamma_{B_s} \gg |\Delta\Gamma_{B_s}|$$

$$r_1 = O(1) \quad , \quad r_2 \lesssim O(10^{-2}) \quad . \tag{51}$$

In conclusion, there is a case for putting some effort into an experimental study of $B^0\bar{B}^0$ mixing, since it offers some hope of shedding new light on the elusive problem of the origin of CP violation.

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